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Optimization of Ram Accelerator Systems Design

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A numerical procedure for optimizing the performance of ram accelerator hypersonic launchers, by variation of the reacting mixture chemistry, is presented. The calculations are performed by a specially developed program that includes a computational fluid dynamics solver for the reacting flowfield solution and an optimization algorithm. The thrust coefficient is taken as the objective function and existing constraints are treated by the exterior penalty method. To illustrate the procedure, optimized solutions are found for a multistage launcher, whose mission is to accelerate projectiles, weighing 0.1 and 100 kg, from 2500 to 9700 m/s. Preliminary investigation of projectile geometric parameters is conducted and shows that they have remarkable effects on the performance of a superdetonative ram accelerator system. Inviscid flow is assumed. The procedure is general and can be used to optimize ram accelerators operating in any of the three established modes: thermally choked, transdetonative, and superdetonative. It also makes possible the estimation of the performance limits for such systems. It is concluded that to obtain realistic solutions, existing limitations and the effect of viscosity should be included in the optimization model. Directions for future research are suggested.

Nomenclature

A	= internal cross-sectional area of the launcher
a	= acceleration
C_f	= thrust coefficient $F/(p_0 A)$
c	= sound speed
D_t	= internal diameter of the launcher
F	= thrust
$f(\boldsymbol{X})$	= objective function
$g_j(\boldsymbol{X})$	= constraint j
L	= length of the launcher
M	= Mach number
m	= mass of the projectile
n_i	= number of moles for species i
p	= pressure
r	= internal radius of the launcher
T	= temperature
t	= time
V	= projectile velocity
X_i	= geometric optimization variables
x	= horizontal coordinate
x_i	= ith optimization variable
y	= radial coordinate
α_1	= half-angle of fore cone
α_2	= half-angle of rear cone

Subscripts

cj	= Chapman–Jouguet condition
f	= final
i	= initial
ig	= ignition
max	= maximum
min	= minimum
opt	= optimal
t	= tube wall of the launcher
0	= freestream condition
$0r$	= representative freestream conditions

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Introduction

THE ram accelerator (RA), sometimes described as a ramjet-in-tube, is a launcher in which projectiles are accelerated to ultrahigh velocities. The projectile is injected, at Mach numbers between 2 and 3, into a barrel filled with a gaseous, reacting mixture at high pressure. The mixture, compressed and heated by shock waves, ignites, and the resulting high pressure on the projectile afterbody generates thrust.

Three overlapping modes of RA operation have been identified as the projectile velocity increases[1]: subdetonative (or thermally choked), transdetonative, and superdetonative. The RA performance in the thermally choked mode can be analyzed by a simple, one-dimensional, streamtube model.[2] All modes can be analyzed by computational fluid dynamics (CFD) methods: The flowfield around the projectile is first solved, then the thrust is calculated by integrating the pressure and the viscosity forces acting, in the flight direction, on the projectile body.

Theory indicates, and experiments have confirmed, that the range of velocities that can be obtained in each mode, using a given mixture, is limited.[1,2] To reach higher velocity, the launcher tube can be divided into several segments having different reacting mixtures, whose operational velocity ranges complete and overlap each other. While doing this, it is possible to choose mixture compositions, section lengths, storage pressures, etc., in such a way as to optimize the system performance.

The optimization of complex systems, such as hypersonic vehicles, crosses many disciplinary boundaries and is still a developing field. Bussing[3] applied an optimization procedure to a generic hypersonic scramjet vehicle. An optimization program, coupled with a nose-to-tail flowfield analysis program and applied to a nozzle design problem, showed that the nozzle thrust can be improved. The steepest descent procedure was used to optimize the geometry of the nozzle that was represented by a quadratic polynomial curve.

A variable-geometry hypersonic waverider vehicle integrated with a scramjet was optimized by O'Neill and Lewis.[4,5] The cruise and acceleration configurations were optimized for maximum specific impulse at an equivalence ratio of one. The Simplex optimization method was used together with a one-dimensional model for the scramjet performance. The nozzle was modeled assuming frozen isentropic flow, and the two-dimensional characteristics method was used. The Simplex method is adequate for nonconstrained problems, but it rapidly becomes inefficient as the number of variables increases.

Baysal and Eleshaky[6] applied sensitivity analysis to optimize the nozzle of a scramjet. The method they used can decrease the number of times the objective function is evaluated in the optimization process, but this technique greatly increases the work of constructing sensitivity analysis models; another shortcoming is that the results of the optimization are less accurate than those of other techniques.

In 1994, Thompson and Riley[7] presented a pattern search method based on the work of Hook and Jeeves, considering an unconstrained optimization and only two design variables. An approximate engineering code was linked with the optimization program to minimize the drag and heatload on the nose of a conical body at hypersonic flight conditions.

In recent years, optimization techniques, such as global optimization, have been used in some research areas, such as truss structure design.[8] Typical techniques are genetic algorithms[9] and simulated annealing (SA) algorithm.[10] These techniques are suitable when dealing with problems that have either disjoint or nonconvex design space, but they usually find the fully converged optimum very slowly.[8] Comparative calculations were made[11,12] that also showed that many more objective function evaluations are needed to obtain convergence using SA than using gradient-based techniques.

The idea of optimizing the performance of ram accelerators was suggested by Hertzberg et al.[1,13] and Hertzberg.[14] However, most of the work was experimental, and an efficient and general numerical procedure to enhance the performance of RA systems is still needed.

Some work in CFD simulation of RA was done by Bogdanoff,[15] Brackett and Bogdanoff,[16] Dyne et al.,[17] Yungster and Bruckner,[18] Yungster et al.,[19] Nusca,[20] and Li et al.[21] In Ref. 17, the analysis of the performance of a superdetonative ram accelerator (SDRA) system, at different mixture equivalence ratios and tube fill pressures, was presented. The impact of projectile geometry on the RA performance was experimentally investigated by Imrich et al.[22]

All of the cited work focused on the simulation of a few specific cases; no general optimization techniques were developed for the analysis of RA systems. These studies gave useful information about the performance of RAs under certain conditions.

An optimization analysis of an external RA system was conducted by Sabean and Lewis.[23] They introduced a two-steps global reaction model, and the geometric parameters of the projectile were taken as optimization variables. A mixture map method was suggested by Elvander et al.[24] to optimize a one-stage thermally choked RA (TCRA) and to analyze the effects of the gas mixture composition on the projectile acceleration. Recently, an optimization procedure based on the variation of the gas mixture composition for multi-staged TCRAs was developed by the authors,[25] which resulted in a more general optimization tool for TCRA systems design. The thrust coefficient of a TCRA has the typical behavior shown in Fig. 1; as will be shown, the behavior of the SDRA thrust coefficient is quite similar.

In the present work, a numerical procedure for optimizing the SDRA performance, by varying the reacting mixture chemistry along the barrel, is presented and demonstrated. Three main algorithms have been developed and linked into a single optimization program (OPTRAM): 1) a CFD code (RAM2D) that solves the Navier–Stokes equations for a reacting flowfield, 2) a chemical kinetics package that calculates the source terms for the species conservation equations, and 3) an optimization procedure (OPTAG) that defines the chemical composition of the mixture yielding the highest performance at a given velocity.

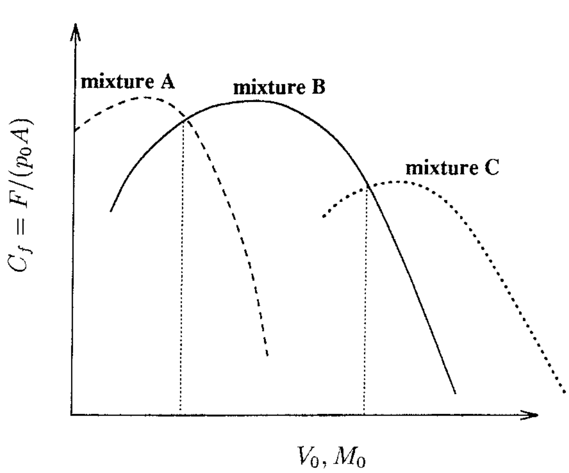


Fig. 1 Typical performance of a TCRA system.

Optimization Algorithms

The general approach to design optimization problems is to use specially developed numerical algorithms to find the parameter values for which a multidimensional objective function (expressing the system performance) assumes extreme values. In most practical cases, a number of constraints are imposed on the solution.

Mathematically, the optimization problem may be stated as follows: Find

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

that minimizes $f(X)$, subject to the constraints $g_j(X) \leq 0$, $j = 1, 2, \ldots, m$, where X is an n-dimensional design vector, $f(X)$ is the objective function, and $g_j(X)$ are the constraints.

When dealing with constrained optimization problems, penalty terms[26] are added to the objective function.

The most popular optimization methods are gradient-based (GB) approaches. The search for the extremum of the objective function starts from an initial guess and proceeds toward the solution, following a path that is a compromise between optimizing the objective function and moving away from each active constraint boundary.

The sequential unconstrained minimization technique (SUMT) has been chosen to translate constrained objective functions to unconstrained ones, by the addition of penalty terms. Two options were available: the interior and the exterior penalty function methods. Rao[26] has discussed and compared these methods with each other. The exterior penalty method was finally selected for use in our research.

The variable metric method, also called the Davidon–Fletcher–Powell method (DFPM) (see Refs. 26 and 27) has been selected in the present optimization procedure. This method is one of the best GB techniques for unconstrained optimization problems with continuous derivatives. When the function does not possess continuous derivatives, Powell's method (see Ref. 26) is used instead.

For a one-dimensional line search of the design cycle, two procedures have been selected. The quadratic interpolation method (QIM)[26] can be efficiently used when the objective function has a smooth surface; for nonsmooth objective functions, the golden section method (GSM)[26] is applied.

Formulation of the RA Optimization Problem

The elements of the RA optimization problem are listed as follows.

1) The objective function $f(X)$ is the negative of the thrust coefficient C_f. The optimization program developed searches for a minimum value. Because we are looking for a maximum, a minus sign is added to the objective function; therefore,

$$f(X) = -C_f = f\left(n_{O_2}, n_{H_2}, n_{\text{diluent}}, p_0, T_0, \ldots\right) \tag{1}$$

2) The optimization variables (the components of vector X) include: a) the number of moles in the reacting mixture (n_{O_2}, $n_{H_2}, \ldots, n_{\text{diluent}}$), b) the initial pressure p_0 and temperature T_0, and c) the geometrical dimensions of the projectile.

3) Typical constraints imposed on the solution may include: a) chemistry, $n_{i,\min} \leq n_i \leq n_{i,\max}$; b) pressure at the tube wall, $p_t \leq p_{\max}$, where $p_{\max}$ is the highest allowable pressure without risking bursting the barrel; and c) geometry.

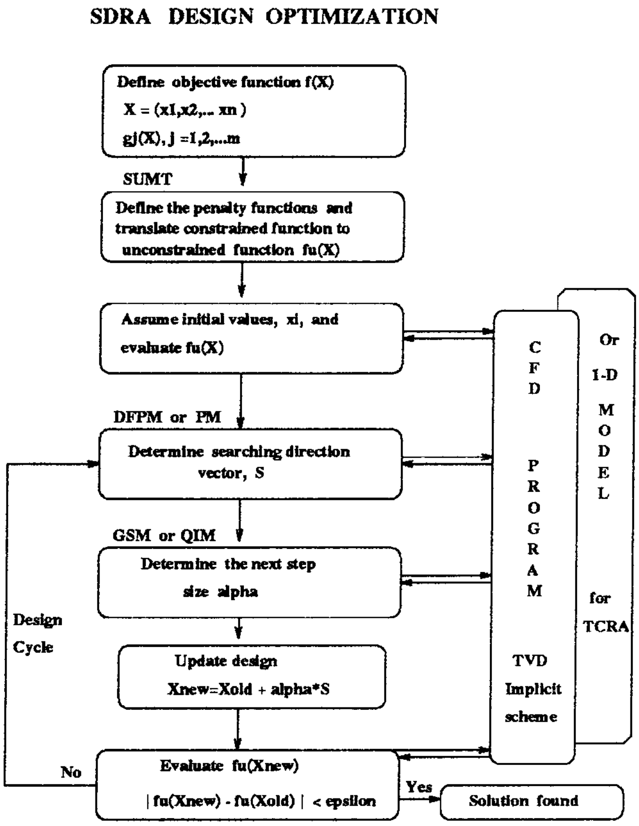


Fig. 2 OPTRAM program flow chart.

4) In most practical cases, some parameters are kept constant during the optimization process; among them are a) the geometry and mass of the projectile, b) the internal diameter of the launcher, and c) the initial temperature and pressure of the gas mixtures.

The flowchart of OPTRAM, given in Fig. 2, illustrates the main steps in a typical RA optimization process.

Numerical Methods

CFD Application

A CFD code, RAM2D, was developed to solve the reacting flowfield around the projectile; a finite rate reaction model was fully coupled with the CFD code. RAM2D can be used to simulate inviscid, laminar, or turbulent flows, with or without combustion.

It is based on the following assumptions.

1) The flow field is two-dimensional/axisymmetric; the three-dimensional effects of the projectile fins and the rails are not considered.

2) A quasisteady situation is assumed to calculate the instantaneous acceleration of the projectile and its displacement.

3) The effect of projectile transition from one mixture to another is neglected.

4) The effect of thermal ablation is neglected, and it is assumed that the major axis of the projectile coincides with the barrel axis (no projectile canting exists).

5) The bore diameter is assumed to be constant along the launcher tube.

The conservation equations set includes a total of 12 variables. In addition, the perfect gas volumetric equation of state is assumed. The equations are formulated for axisymmetric, body-fitted coordinate systems and nondimensional variables and solved by established numerical methods. Because implicit schemes can be run with much larger time steps than explicit schemes, the lower–upper symmetric Gauss–Seidel factored implicit algorithm[28] and the Yee/Harten total variation diminishing TVD scheme[29] are used in the present work. The implicit diagonal matrix for the source terms suggested by Eberhardt[30] was modified.[12]

Chemical Reaction Model

The chemical kinetics model quantifies the forward and backward reaction rates for a set of elementary reactions representing the combustion process. A relatively simple model for H_2/O_2/diluent mixtures, consists of 8 elementary reactions between 7 species. This model was used by Spiegler et al.[31] and then modified by Evans and Schexnayder[32] and applied to simulate RA systems.[18,19] The 8 reactions/7 species model may be less accurate in simulating the ignition process in comparison with the 25 reactions/12 species system proposed by Evans and Schexnayder, but once ignition occurs, good agreement is obtained between the results from the two models.

Benchmark tests were conducted to validate the RAM2D program, which showed good agreement with results published in the literatures.[12] The grid effect on the pressure distribution along the projectile surface was investigated for the geometry of Fig. 3 and $V_0 = 5000$ m/s, $p_0 = 20$ atm, $T_0 = 300$ K, with $O_2/2H_2/2.8He$ mixture. Three grids, with different numbers of computational cells, were considered: 1) 91 × 16, 2) 101 × 22, and 3) 121 × 32.

The results (Fig. 4) show that the calculated pressures along the fore and aft cones are practically identical. Differences exist along the cylindrical section of the projectile, which does not contribute to the thrust. As a result, the effect of the grid structure on the value of the thrust coefficient is quite small (less than 1%). In consequence, the 101 × 22 grid was considered adequate and was selected for further calculations.

The convergence history of the numerical solution for reacting flow was also investigated for three velocities: 2500, 5000, and 9000 m/s ($M_0 = 6.7$, 7.33, and 10.6, respectively). Large time steps [Courant–Friedrichs–Lewy number (CFL) = 3.0] were used for the first 500 iterations, when the mixture was not allowed to react; therefore, the inert flow solution converged rapidly. After that, the reaction model was made operational, and the time step was decreased (CFL = 0.3) to avoid divergence. The convergence history showed that after some initial oscillations due to the change in time step and the initiation of the chemical reaction, steady convergence is

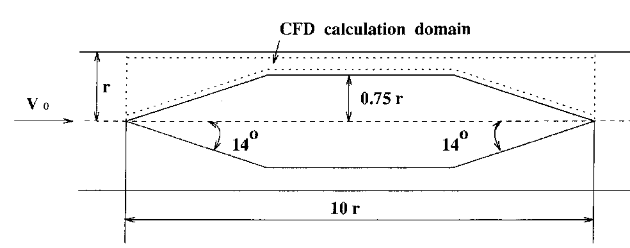


Fig. 3 RA geometry.

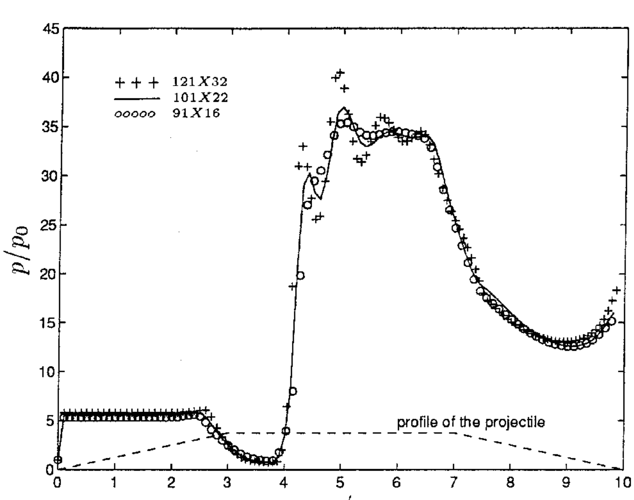


Fig. 4 Comparison between the calculated pressures on the projectile surface using different grids.

achieved in all cases. The number of iterations needed for three-orders-of-magnitude convergence varies from about 15,000 for $V_0 = 2500$ m/s to about 7,000 for $V_0 = 9000$ m/s. In the subsequent calculations, it was found that this range is representative for all cases investigated. A fully converged solution ($L_2 = 10^{-7}$) takes between 2 and 5 h CPU time on a Cray-J932 computer.

SDRA System Optimization

General Considerations

Assuming that every mixture selected by the optimization program has adequate ignition and combustion properties, it is theoretically possible to define, by continuous variation of the mixture chemistry, an ideal optimized solution, as schematically shown in Fig. 5: Each point along the ideal optimal $C_f = f(V_0)$ curve represents the optimized mixture for the respective velocity V_{0r}.

In practice, the launcher is divided in a finite (and reasonable) number of sections, each one containing an optimized mixture for a representative velocity. The length of each section is defined by the intersection points of adjacent $C_f = f(V_0)$ curves. Some performance is lost when compared to the ideal optimization case (the infinite number of sections), and the difference increases as the number of sections decreases. The designer must find a compromise that balances the loss in performance against system simplicity.

Optimization for the Representative Velocities

The RA optimization procedure will be now demonstrated for the eight representative velocities. The thrust coefficient is chosen as the optimization objective function. The projectile geometry (shown in Fig. 3) is similar to the shape used at French-German Institute at Saint Louis (ISL).[33] The internal radius of the launcher is $r = 19$ mm. The launcher will be divided in eight sections, each containing an $O_2/H_2/He/N_2/Ar$ gas mixture optimized for the following representative velocities: $V_{0r} = 2500$, 3000, 4000, 5000, 6000, 7000, 8000, and 9000 m/s. The initial pressure and temperature in all section are $p_0 = 20$ atm and $T_0 = 300$ K. The inviscid flow assumption will be applied in the CFD simulation.

The numbers of moles of the components O_2, H_2, He, N_2, and Ar are the design variables. The constraints are represented by limitations on the number of moles for each component in the mixture.

We can express the optimization goal as follows. Maximize:

$$C_f = f\left(n_{O_2}, n_{H_2}, n_{N_2}, n_{Ar}, n_{He}\right) \qquad (2)$$

subject to the constraints

$$n_{i,\min} \le n_i \le n_{i,\max} \qquad (i = 1, 2, \ldots, 5) \qquad (3)$$

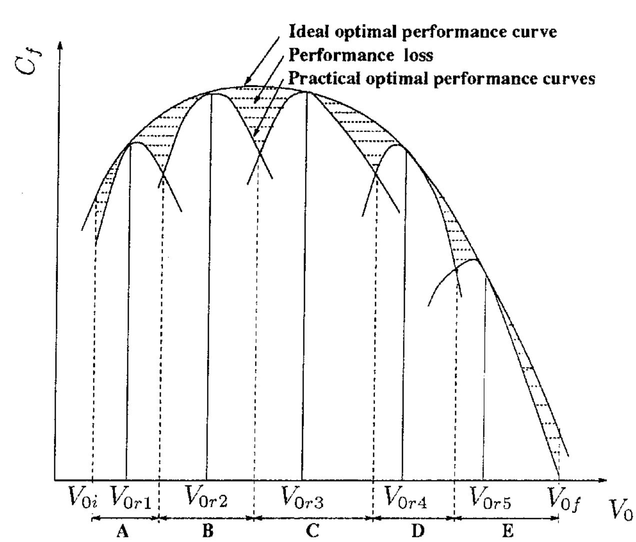


Fig. 5 Ideal vs practical RA performance optimization.

Table 1 SDRA optimization

Velocity (m/s)	Optimized mixtures	Mach	c (m/s)	V_{cj} (m/s)	C_f
2500	$O_2 + 2.05H_2 + 1.88N_2 + 3.05Ar$	6.70	373	1890	5.61
3000	$O_2 + 2.20H_2 + 0.19N_2 + 1.56Ar$	7.21	416	2270	7.10
4000	$O_2 + 2.11H_2 + 0.50He + 0.42Ar$	7.79	514	2840	7.28
5000	$O_2 + 2.112H_2 + 2.81He$	7.33	682	3670	6.62
6000	$O_2 + 2.39H_2 + 3.29He$	8.48	707	3780	5.81
7000	$O_2 + 2.52H_2 + 4.62He$	9.40	745	3900	4.87
8000	$O_2 + 2.94H_2 + 5.61He$	10.3	776	3960	3.09
9000	$O_2 + 4.35H_2 + 9.73He$	10.6	853	3860	1.17

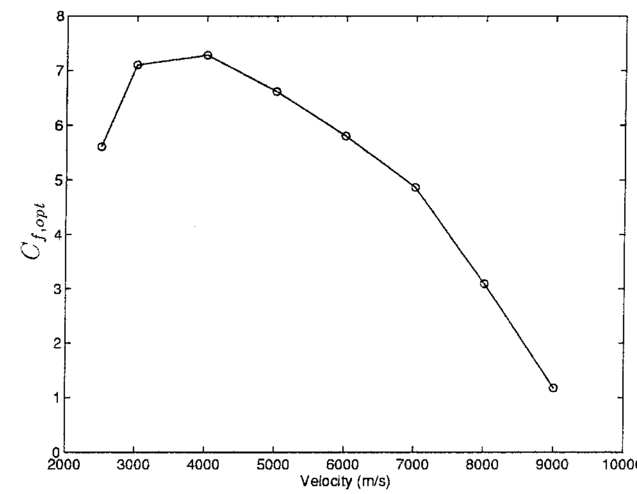


Fig. 6 $C_{f,\mathrm{opt}}$ vs V_{0r}.

where the minimum values, $n_{i,\min}$, are equal to zero; the $n_{i,\max}$ are chosen to be 15 moles. The initial values for the design variables are chosen so as to satisfy the condition $5 < M_0 < 11$. The reason for this is to ensure that the searching procedure starts in the superdetonative range. The calculation grid (101×22) is chosen within the calculation domain shown in Fig. 3. In the following optimization calculations, the lower constraints for the molar numbers are set to be zero or very small values. To avoid possible negative values of $n_{i,\min}$, a larger penalty factor is included into the penalty terms.

Optimization Results

The results of C_f optimization for the eight representative velocities are summarized in Table 1 and graphically shown in Figs. 6–9.

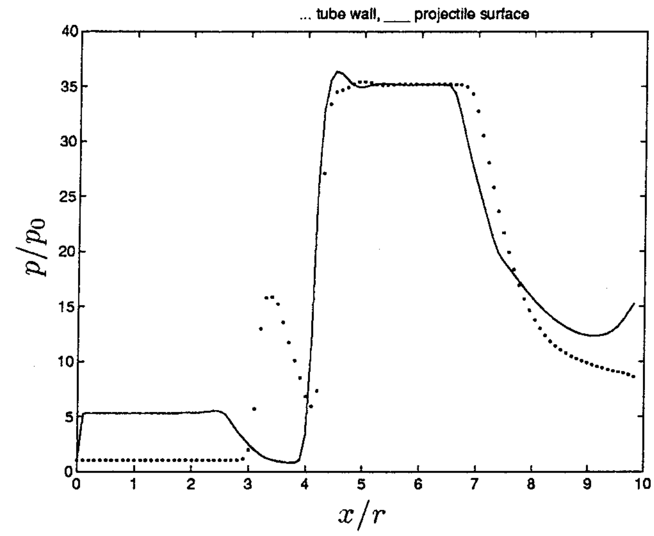
The following remarks may be made.

1) The RA operates in the superdetonative mode at all velocities, as V_{0r}/V_{cj} varies between 1.32, at $V_{0r} = 2500$ m/s, and 2.33 at $V_{0r} = 9000$ m/s (see Table 1).

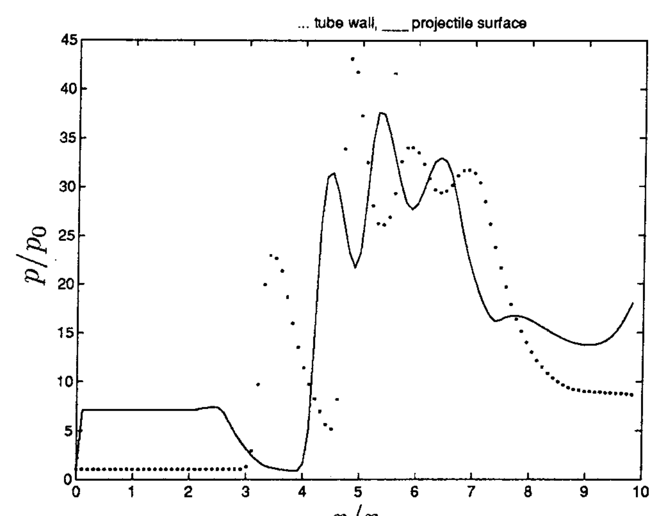
2) The mixtures selected by the optimization procedure are fuel rich ($n_{H_2}/n_{O_2} > 2.0$), especially at higher velocities. Helium is added, instead of argon, at velocities of 5000 m/s and higher. The increasing predominance of low-molecular-weight components leads to mixtures having higher and higher sound velocities. As a result, the Mach number of the accelerating projectile increases at a lower rate than its velocity: $M_{v_0=9000}/M_{v_0=2500} = 1.58$, as compared to $9000/2500 = 3.6$ (see Table 1).

3) As the velocity increases from 3000 to 9000 m/s, the average pressure on the nose cone, which generates drag, increases from $5p_0$ to $11p_0$, while the average pressure on the tail cone, which generates thrust, decreases from $15p_0$ to $13p_0$ (see Fig. 7). This explains the downward trend of the performance curve (Fig. 6) at high velocities.

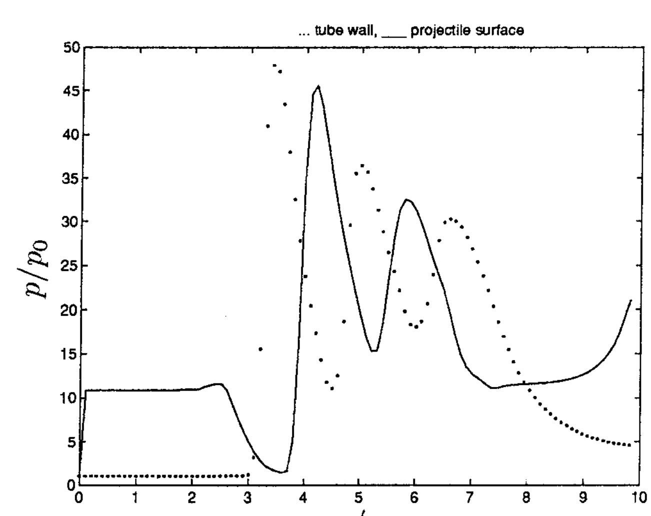
4) The peak pressure on the launcher wall increases with V_0 and reaches $48p_0$, when $V_0 = 9000$ m/s (Fig. 7). In our case ($p_0 = 20$ atm), maximum $p_t = 960$ atm. If higher initial pressures are contemplated as a direct and simple way of increasing the thrust, the constraint $p_t < p_{\max}$ should be added to the optimization procedure.



a) V_0 = 3000 m/s and M_0 = 7.2

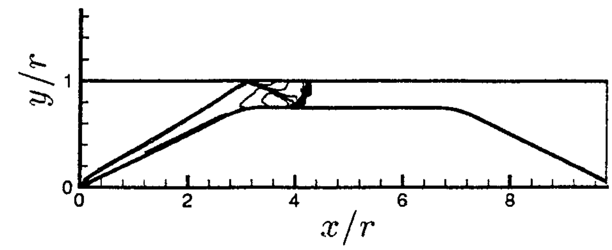


b) V_0 = 6000 m/s and M_0 = 8.86

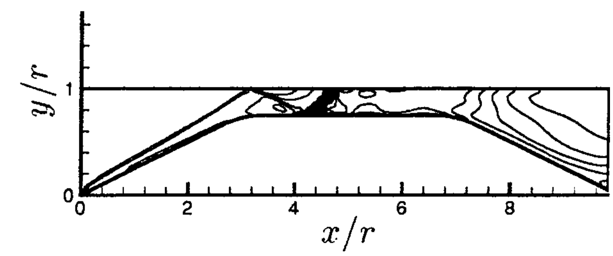


c) V_0 = 9000 m/s and M_0 = 10.8

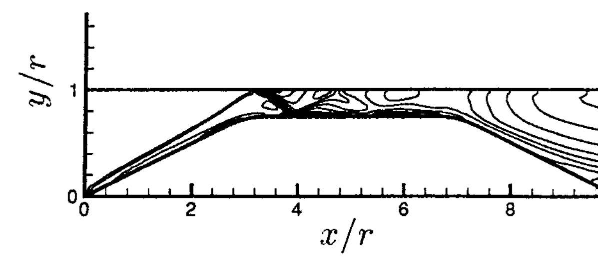
Fig. 7 Pressure ratio (p/p_0) distribution.



a) V_0 = 3000 m/s and M_0 = 7.2



b) V_0 = 6000 m/s and M_0 = 8.86



c) V_0 = 9000 m/s and M_0 = 10.8

Fig. 8 Temperature (T/T_0) contours.

5) The combustion front, which can be observed in Figs. 8 and 9, is to be fully coupled with the first ($V_0 = 9000$ m/s) or the second ($V_0 = 3000$ and 6000 m/s) reflected shock wave. This is characteristic to detonation waves, which are combinations of shock waves and supersonic combustion.

6) The limit velocities can be estimated in terms of V_{cj}. The highest velocity is $2.51 V_{cj}$ (shown in Fig. 10) for the mixture optimized at 9000 m/s (see Table 1) and $2.24 V_{cj}$ (not shown in the figures) for the mixture optimized at 3000 m/s.

Determining the Design Curve

For the design of an optimized RA system, the relation between flight velocity and the length of the eight sections of the launcher can be easily determined by using the preceding results. This is done as follows.

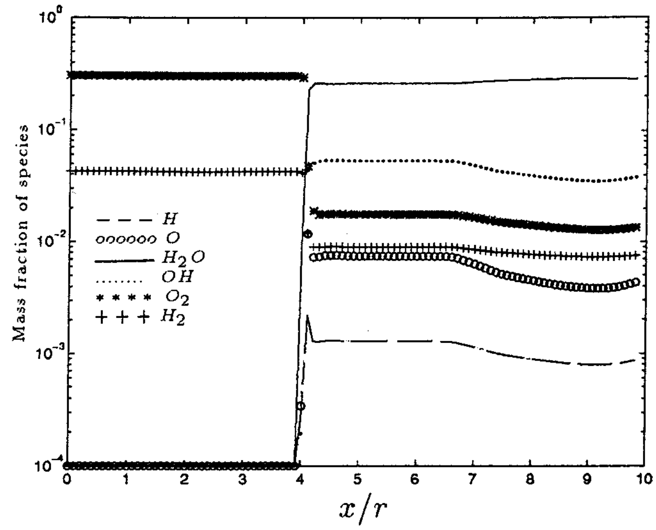
First, C_f values are calculated for several V_0 values on both sides of the representative velocities, using the respective optimized mixtures.

Next, the calculated C_f values are plotted vs V_0, and the range of the velocities for each mixture is determined by neighboring curves intersections, which are also the segments of the multistage launcher (see dashed lines in Fig. 10). The abrupt increase in the C_f values at low velocity for some representative curves (Fig. 10) is attributed to the chemical reaction kinetics model used in our calculation and/or to the assumption of ignition temperature $T_{ig} = 900$ K we chose.

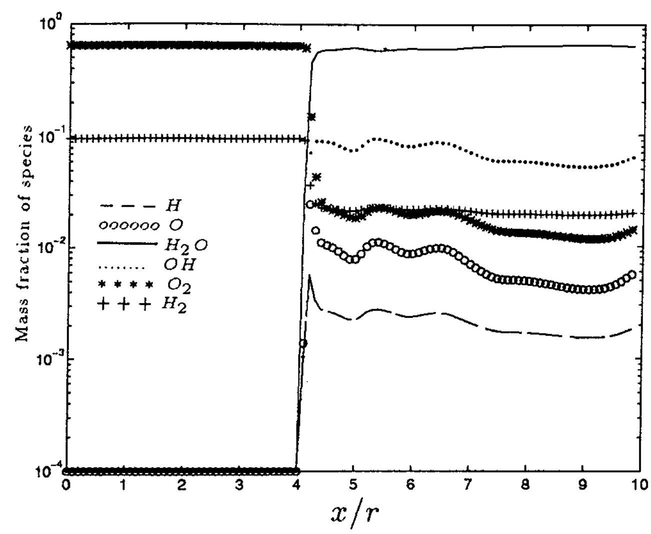
Using the $C_f = f(V_0)$ curve of Fig. 10, the length of the launcher may be calculated as follows:

$$\frac{\mathrm{d}V_0}{\mathrm{d}x} = \frac{\mathrm{d}V_0}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{a}{V_0} = \frac{F}{V_0 m} = \frac{C_f p_0 A}{V_0 m} \tag{4}$$

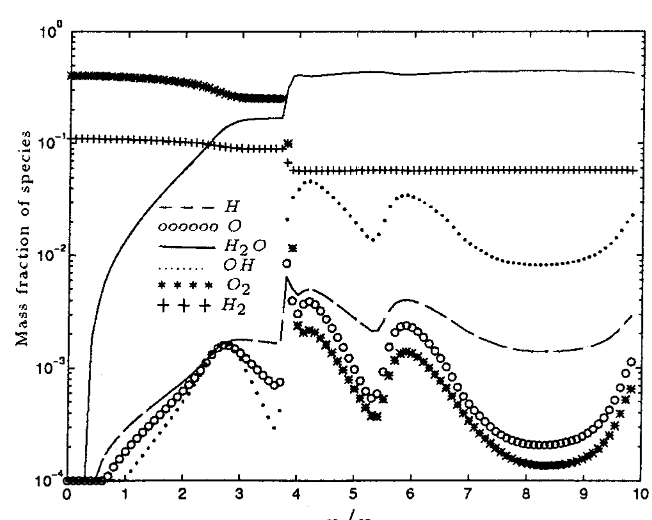
$$\mathrm{d}x = \frac{V_0 m \,\mathrm{d}V_0}{C_f p_0 A} \tag{5}$$



a) V_0 = 3000 m/s and M_0 = 7.2



b) V_0 = 6000 m/s and M_0 = 8.86



c) V_0 = 9000 m/s and M_0 = 10.8

Fig. 9 Species mass fraction distribution along the projectile surface.

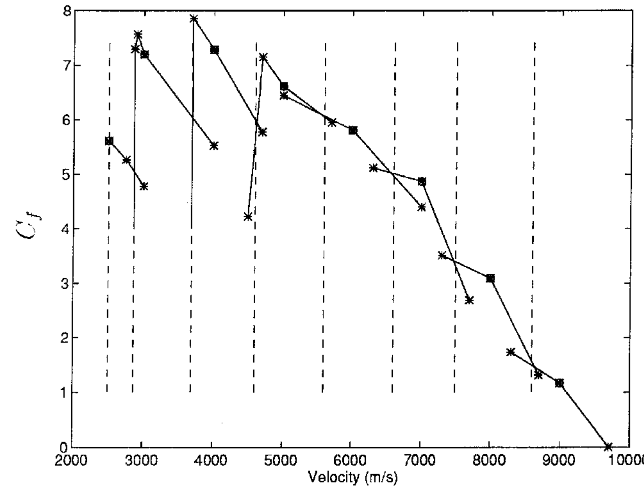


Fig. 10 Performance curves and intersection points.

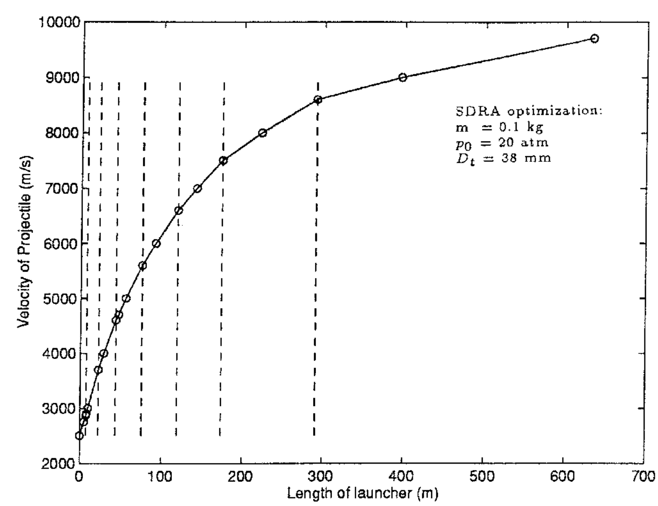


Fig. 11 Optimization results: velocity as a function of launcher length (m = 0.1 kg, p_0 = 20 atm, D_t = 38 mm).

The launcher length is obtained by integration,

$$L = \int_{V_{0i}}^{V_{0f}} \mathrm{d}x = \frac{m}{p_0 A} \int_{V_{0i}}^{V_{0f}} \frac{V_0}{C_f} \mathrm{d}V_0 \tag{6}$$

or, in finite difference formulation,

$$L = \frac{m}{p_0 A} \sum_j \frac{V_{0j}}{C_{fj}} \Delta V_{0j} \tag{7}$$

From Eq. (6) or (7), we can obtain the design curve for the optimal design of a ram accelerator system for the given missions.

Missions

We chose a projectile with $m = 0.1$ kg and $V_{0i} = 2500$ m/s. The parameters r, p_0, and T_0 are the same as earlier calculations in the optimization of the representative velocities. The design curve is shown in Fig. 11 and is capable of designing two kinds of missions: maximum velocity and minimum length of launcher. According to the curve, for a given length of launcher with 400 m, the highest final velocity 9000 m/s can be achieved; for the given final velocity of 9700 m/s, the shortest length of launcher is 640 m long. These results are valid for any $m/p_0/A$ combination having the value $0.1/20/11.34 = 4.4E-4$. Results for other cases may be obtained by scaling; thus, the length of a launcher needed to accelerate a 100-kg projectile using a 500-mm bore and a storage pressure of

100 atm is estimated to be 740 m (for a final velocity of 9700 m/s) or 460 m (for a final velocity of 9000 m/s).

Optimization of the Projectile Geometry

The geometric parameters of the projectile, X_1–X_5 (see Fig. 12), which were considered constant in the preceding section, will now be added as variables of the objective function and the problem of optimizing the SDRA system (reacting mixture + projectile geometry) will be addressed, for three representative velocities: 3000, 6000, and 9000 m/s. The initial parameters T_0 and p_0 are the same as in the preceding section. The results are expected to provide some insight on the contribution of an improved projectile geometry to the system performance of an SDRA system.

The optimization problem to be solved is

$$C_f = f\left(n_{O_2}, n_{H_2}, n_{N_2}, n_{Ar}, n_{He}, X_1, X_2, \ldots, X_5\right) \quad (8)$$

with the design variables subject to the constraints

$$n_{i,\min} \le n_i \le n_{i,\max} \qquad (i = 1, 2, \ldots, n_s) \quad (9)$$

and

$$X_{j,\min} \le X_j \le X_{j,\max} \qquad (j = 1, 2, \ldots, 5) \quad (10)$$

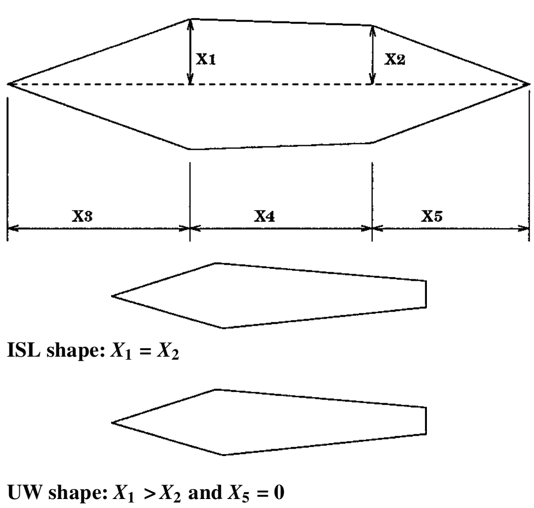


Fig. 12 Projectile geometric parameters.

The constraints for n_i are the same as in the preceding section; X_j are nondimensional parameters, normalized by the internal radius of the launcher. In the present example, the constraints are $0 \le X_j \le 0.85$ (for $j = 1, 2$) and $0 \le X_j \le 10$.

The results of the system optimization are summarized in Table 2 and shown in Fig. 13.

The following remarks may be made.

1) The optimization including geometric parameters result in possibly higher thrust coefficient C_f values than the optimization of reacting mixtures only. For the representative velocities 3000, 6000, and 9000 m/s, the C_f values increase about 38, 81, and 365%, respectively (see Tables 1 and 2).

2) As the representative velocity increases, the optimized projectile configuration becomes more and more slender (see Fig. 13). Both the fore cone half-angle α_1 and the aft cone half-angle α_2 decrease from about 14.5 to about 9.5 deg, while the velocity changes from 3000 to 9000 m/s.

3) The X_1 value reaches the upper constraint (0.85), and the X_2 value varies between 0.78 and 0.85. These variables, among other parameters, affect the flow choking and unstart. However, for the inviscid flow assumption herein, the effects of the flow choking can only be analyzed approximately.

Discussion

The mission defined for the example, although challenging from the required performance point of view, represents a relatively simple case (inviscid flow, fixed projectile geometry, fixed storage pressure, etc.). Even so, it took about 1600 CPU h on a CRAY J932 computer system to obtain a solution. The required computer time is directly proportional to the fineness of the grid (101 × 22 in our example), the number of unknowns to be solved (12), the number of sections (8), the number of optimization cycles (70), and the number of iterations needed to achieve adequate convergence (7,000–15,000). It can be reduced to more acceptable levels mainly by diminishing the number of the representative velocities and by adopting engineering criteria for convergence.

A general design curve, suggested for the practical design of RA, may be used directly for two kinds of missions: 1) maximize the final velocity under given initial velocity and the launcher length and 2) minimize launcher length under the given initial and final velocities. The design procedure is simple and suitable to the design of the RA system in any mode.

The eight representative velocities are chosen to optimize the thrust coefficients for the typical geometry of the superdetonative ram accelerators; the results show that the highest velocity 9700 m/s

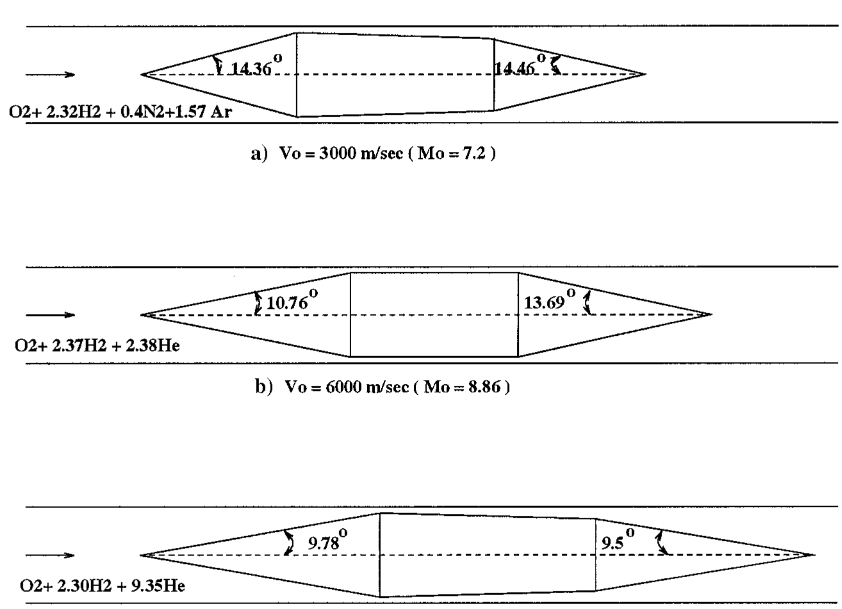


Fig. 13 Optimized geometries for different representative velocities.

Table 2 SDRA optimization including geometric parameters

Velocity (m/s)	Optimized mixtures	Mach	V_{cj} (m/s)	C_f	α_1 (deg)	α_2 (deg)	X_1	X_2	X_3	X_4	X_5
3000	$O_2 + 2.32H_2 + 0.40N_2 + 1.57Ar$	7.20	2258	9.80	14.4	14.5	0.85	0.805	3.32	4.02	3.12
6000	$O_2 + 2.38H_2 + 2.38He$	8.86	3673	10.5	10.8	13.7	0.85	0.85	4.47	3.21	3.49
9000	$O_2 + 3.30H_2 + 9.35He$	10.8	3940	5.44	9.78	9.50	0.85	0.78	4.93	4.42	4.66

is achieved, and the possible range of velocities are between $2.24V_{cj}$ and $2.51V_{cj}$. The maximum velocities, obtained in the current study, may be taken as the upper limits of the superdetonative RA under the given conditions.

The viscosity affects the RA performance in several ways: 1) It generates friction drag. 2) Because of boundary-layer development, the effective port area (between the projectile and the internal wall of the launcher) decreases, thus promoting flow choking and unstart. 3) Premature ignition on the forward cone surface becomes possible, due to high static temperatures in the boundary layer. This should result in lower thrust values and narrower ranges of operational velocities. Clearly, to reach realistic solutions, viscosity must be included in the equations.

The effect of an optimized projectile geometry is also worth investigating. The increase in C_f, due to the geometry optimization, is remarkable, especially at the high end of the velocity range ($V_0 = 6000$ m/s or higher). Optimization calculations show that a slender shape with fore cone + cylinder body + rear cone geometry is favored for an SDRA system. Although a variable projectile geometry is not practical, the possibility exists that an optimized geometry can be defined for a certain range of velocities.

Finally, the significance of including, or not, the existing practical constraints in the optimization model must be stressed. Ignoring such limitation as, for example, the maximum allowed wall pressure or the highest acceleration limit for acceleration-sensitive payloads will lead to nonrealistic solutions. In the specific case of pressure limitation, the definition of the objective function must be changed from $f(X) = -C_f$ to $f(X) = -p_0 C_f$.

It should be emphasized that the OPTRAM program, in its present form, is perfectly capable of simulating and analyzing the aforementioned effects and is currently being used for this purpose.

Conclusions

A procedure and a computer program (OPTRAM) for the optimization of RA systems performance have been developed and demonstrated for the case of an SDRA launching payload at a muzzle velocity of 9700 m/s. The program can handle RAs operating in any mode (thermally choked, transdetonative, and superdetonative); it can also solve non-connected fluid dynamics and optimization problems. The optimization algorithm allows for various definitions of the objective function and for optional constraints. The optimizations in the example are achieved by mixture variation, and further analyses including geometric parameters indicate that even higher thrust coefficients can be obtained.

Reviewing the results obtained until now, the following conclusions may be drawn: 1) The OPTRAM program is a versatile and reliable tool for optimizing a wide spectrum of RA systems. 2) For realistic results, all existing constraints should be included in the optimization model. 3) Viscosity effects should be accounted for, especially when dealing with small projectiles flying at velocities higher than 6000 m/s. 4) The impact of an optimized projectile geometry should be assessed. 5) Ways must be found to alleviate the excessive computer time problem.

In addition to the on-going activity already mentioned, future efforts should concentrate on the following subjects: 1) validating the OPTRAM program by comparing its results to experimental data, 2) introducing an improved chemical kinetics model for a better simulation of the ignition and reaction process, 3) investigating the effect of projectile transition from one mixture to another under consideration of viscous flow, and 4) developing a parallel version of the OPTRAM program, for a significant reduction of the wall-clock time.

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